Total No. of Questions : 05] [SET-A] [Total No. of Printed Page : 02

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

Final Year – Fourth Semester

Special Function – II

4M.Sc. 3

Time : 3 Hours

Max Marks : 70

Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) Find the generating function of the Hermit polynomials H_n(x).

(b) Express H(x) = x^4+ $2x^3$+$2x^2$-x-3, in terms of Hermite's polynomials.

(c) Show that

$2xH_n(x) = 2nH_{n-1}(x) + /H_{n+1(x).}$

Q.2. (a) Find the Bateman's generating relation.

(b) Prove that the orthogonal property of $H_n(x)$

$\int_{-\infty}^{\infty} exp(-x^2)H_n(x)H_m(x)dx = 0, if\ m \neq n$

(c) Show that

$\int_{-\infty}^{\infty} x^2 e^{-x}\{H_n(x)^2\}^2 dx = \sqrt{\pi}2^n \lfloor n \left(n+\frac{1}{2}\right)$

Q.3. (a) Find the Rodrigue's formula for Laguerre polynomial.

(b) Prove that the recurrence relation

$x\text{L}_n^1(x) = n\text{L}_n(x) - n\text{L}_{n-1}(x)$

(c) Show that

$$\int_{\sigma}^{\infty} e^{-st}\text{L}_n(t)dt = \frac{1}{s}\left(1-\frac{1}{s}\right)^n$$

Q.4. (a) Define generalized laguerre polynomial.

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Q.4. (a) Define generalized laguerre polynomial.

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(b) Show that the first differential recurrence relation

$$D\left\{L_n^{(\propto)}(x)\right\} = D\left\{L_{n-1}^{(\propto)}(x)\right\} - L_{n-2}^{(\propto)}(x)$$

(c) Find the Rodrigue's formula for generalized Lagurre polynomial.

Q.5. (a) Prove thate for Jacobi Polynomial-

$$P_n^{(\alpha,\beta)} = \frac{(-1)^n (1-x)^{-\alpha} (1+x)^{-\beta}}{2^n \lfloor n} D^n\left[(1-x)^{n+\alpha}\, (1+x)^{n+\beta}\right]$$

(b) Prove that

$$\int_{-1}^{1} (1-x)^{\alpha}\, (1+x)^{\beta} \left[P_n^{(\alpha,\beta)}(x)\right]^2 dx$$

$$= \frac{2^{\alpha+\beta+1} \sqrt{(1+\alpha+n)}\, \sqrt{(1+\beta+n)}}{\lfloor n (1+\alpha+\beta+2n) \sqrt{(1+\alpha+\beta+n)}}$$

(c) Explain Jacobi polynomials.

------x------

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