Total No. of Questions : 05] [SET-A] [Total No. of Printed Page : 02

FINAL EXAMINATION – JULY 2017
MASTER OF SCIENCE (M.Sc. MATHEMATICS)
Final Year – Fourth Semester
Functional Analysis – II

4M.Sc. 1

Time : 3 Hours **Max Marks : 70**
Min. Marks : 25

Note :- Solve any two parts from each question. All questions carry Equal marks.

Q.1. (a) State and Prove open mapping theorem.

(b) Let B and B' be Banach spaces and let T be a linear transformation of B into B'. Then T is a continuous mapping if and only if its graph is closed.

(c) State and prove Hahn-Banach theorem.

Q.2. (a) Define Hilbert Space. Write its properties with proof.

(b) If x and y are any two vectors in a Hilbert space H, then $|(x,y)| \leq \|x\|.\|y\|$.

(c) If x and y are any two vectors in a Hilbert space, then-

(i) $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

(ii) 4(x,y)= $\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$

Q.3. (a) If S, S_1, S_2 are non-empty subsets of a Hilbert space H, then prove the following:-

(i) $S \cap s^1 = \{0\}$

(ii) $S_1 \subset S_2 \Rightarrow S_2^1 \subset S_1^1$

(iii) $S \subset S^{11}$

(b) State and prove projection theorem.

(c) If $\{e_i\}$ is an orthonormal set in a Hilbert space H, then $\sum |x, e_i|^2 \leq \|x\|^2$ for every vector x in H.

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Q.4. (a) Show that the mapping $\psi: H \rightarrow H^*$ defined by $\psi(y) = f_{y\,,}$where $f_y(\mathrm{x}) = (\mathrm{x}, \mathrm{y}) \forall \mathrm{x} \epsilon \mathrm{H}$ is one-to-one, onto, additive but not linear and an isometry.

(b) State and prove Riesz representation theorem for linear functional on a Hilbert space.

(c) If H is a Hilbert space, then H is reflexive.

Q.5. (a) Show that the adjoint operation $T \rightarrow T^*$ on B(H) has the following properties-

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$

(ii) $(\alpha\, T)^* = T_2^*. T_1^*$

(iii) $\|T^*\| = \|T\|$

(b) Show that the adjoint operation is one-to one onto as a mapping of B (H) into itself.

(c) If T is an operator on a Hilbert space H, then $\left(T_{x,} x\right) = 0$ for all x in H $\Leftrightarrow T = 0$.

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