Total No. of Questions : 05] [SET-A] [Total No. of Printed Page : 02

FINAL EXAMINATION – JULY 2017
MASTER OF SCIENCE (M.Sc. MATHEMATICS)
Final Year – Third Semester
Special Function – I

3M.Sc. 3

Time : 3 Hours **Max Marks : 70**
Min. Marks : 25

Note :- Solve any two parts from each question. All questions carry Equal marks.

Q.1. (a) Define (Any two)

(i) Special function
(ii) Infinite series
(iii) Orthogonal polynomial

(b) Find the Euler's product $\Gamma(Z)$

(c) Show that $\int_{0-}^{\infty} \cos x^2 \, dx = \frac{1}{2}\sqrt{\pi/2}$

Q.2. (a) Give one introduction of Hypergeometric functions.

(b) By applying suitable transformations to the hypergeometric equation, obtain the identity

$(1-Z)^{-a}$ F $\begin{bmatrix} a, c-b\,; & \frac{-z}{1-z} \\ c\;\;; & \end{bmatrix}$ = F [a,b; c;z]

(c) Define Integral representation of f [a, b; c,z]. The complete elliptic integral of first kind being

$$K = \int_0^{\frac{1}{2}\pi} \frac{d\phi}{\sqrt{1-k^2 \sin^2\phi}},$$

Show that k = $\frac{1}{2}\pi\, F\,(\frac{1}{2},\frac{1}{2}\,;\,1\,;\,k^2)$

Q.3. (a) Explain clearly the generalized Hypergeometric Function.

(b) State and prove Saalschutz theorem.

(c) State and prove Dixon's theorem.

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(b) State and prove Ramanujan's theorem.

(c) Prove that

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Q.5. (a) Show that

J_n (-z) = $(-1)^n J_n$(z) for positive or negative integer n

(b) prove that

(i) $J_{1/2}(z) = (\frac{2}{\pi z})^{1/2} \sin z$

(ii) $J_{-1/2}(z) = (\frac{2}{\pi z})^{1/2} \cos z$

(c) Explain Neumann polynomials and series.

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