

FINAL EXAMINATION – JULY 2017  
 MASTER OF SCIENCE (M.Sc. MATHEMATICS)

Final Year – Third Semester  
 Integral Transform - I

3M.Sc. 2

Time : 3 Hours

Max Marks : 70

Min. Marks : 25

Note :- Solve any two parts from each question. All questions carry Equal marks.

- Q.1. (a) State and prove Laplace inverse convolution theorem.  
 (b) State and prove Laplace Heiri-Side's expansion theorem.  
 (c) Find the Laplace transform of H(t) defined as

$$H(t) = \begin{cases} t + 1, & 0 \leq t \leq 2 \\ 3, & t > 2 \end{cases}, \text{ Also determine } L\{H'(t)\}.$$

- Q.2. (a) Using Laplace transforms. Find solution of  $y'' + 25y = 10 \cos 5t$ , where  $y(0) = 2, y'(0) = 0$ .  
 (b) Solve  $(D^2 + 1)x - Dy = 1$   
 $Dx + (D^2 + 2)y = 0$   
 With  $t > 0; x = 0, Dx = 0 = y = Dy$ , when  $t = 0, x$  and  $y$  both being function of  $t$ .  
 (c) State and prove the application of Laplace transform to integral Savit equation.

- Q.3. (a) State and prove the application of electrical circuits to Laplace transform.  
 (b) Solve the boundary value problem

$$\frac{\partial^2 4}{\partial t^2} = a^2 \frac{\partial^2 4}{\partial x^2}, (t > 0, x > 0), \text{ where } 4(x, 0) = 0, x \geq 0, 4t \quad (x, 0) = 0, x > 0. \\ 4(0, t) = t, \lim_{x \rightarrow \infty} 4(x, t) = 0, t \geq 0$$

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(c) State and prove Heat Conduction equation of Laplace transform.

Q.4. (a) Obtain a Series of Sines and Cosines of multiples of  $x$  which will represent  $f(x)$  in the interval  $-\pi < x < \pi$  when

$$f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ \frac{\pi x}{4}, & 0 < x < \pi \end{cases} \text{ and hence deduce that } \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots.$$

(b) State and prove Modulation theorem of Fourier transform.

(c) State and prove Fourier Integral theorem.

Q.5. (a) State and prove Parseval's identity for Fourier transform.

(b) Show that  $\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + 1} d\lambda = \frac{\pi}{2} e^{-x}, x \geq 0$ .

(c) State and prove inversion formula of Sine Transform.

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