Total No. of Questions : 05] **[SET-A]** **[Total No. of Printed Page : 02**

FINAL EXAMINATION – JULY 2017
MASTER OF SCIENCE (M.Sc. MATHEMATICS)
Final Year – Third Semester
Functional Analysis – I

3M.Sc. 1

Time : 3 Hours **Max Marks : 70**
Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) Define normed linear space. Show that the linear space R^n and C^n of all n-tuples $x = x_1, x_2, \dots x_n$ of real and complex numbers are Normed Linear space under the norm

$$\|x\| = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2}$$

(b) Show that a normed linear space N is a Banach space iff every absolutely summable series in N is summable.

(c) State and prove Holder's inequality.

Q.2. (a) Let P be a real number such that $1 \leq P < \infty$ and denote by ℓ_p^n the space of all n-tuples $x = (x_1, x_2, \dots x_n)$ of scalars. Show that ℓ_p^n is a Banach space under the norm

$$\|x\|_p = \left[\sum_{i=1}^{n} |x_i|^p \right]^{1/p}$$

(b) Let p be a real number such that $1 \leq P \leq \infty$ and let ℓ_p denote the space of all sequences $x = \{x_1, x_2, \text{-----} x_n \text{----}\}$ of scalars such that $\sum_{n=1}^{\infty} |x^n|^p < \infty$. show that ℓ_p is complete under the norm

$$\|x\|_p = \left[\sum_{n=1}^{\infty} |x_n|^p \right]^{1/p}$$

(c) For i ≤ P < ∞ show that ∟p space is complete.

Q.3. (a) Define subspace and quotient space. Let M be a closed linear subspace in a normed linear space N. For each coset x + m in the quotient space N/M we define.

$$\|x + M\| = \inf \{ \|x + m\| : m \in M \}$$

Than $\|x + M\|$ is a norm on N/M and thus N/M is a normed linear space.

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(b) Define continuous linear transformation. Let N and N' be normed linear spaces and let T be a linear transformation of N into N'. Then the inverse T^{-1} exists and is continuous on its domain of definition if and only if there exist a constant m>o such that

$$m\|x\| \le \|T(x)\| \text{ for all } x \in N$$

(c) Let N and N' be normed linear space and let T be a continuous linear transformation of N into N' If M is the kernel (null space) of T, show that T induces a natural linear transformation T' of N/M into N' and that

$$\|T'\| = \|T\|$$

Q.4. (a) Let N be a normed linear space and suppose two norms $\|.\|_1$ *and* $\|.\|_2$ are defined on N Then these norms are equivalent if and only if there exists positive real numbers m and M such that

$$m\|x\|_1 \le \|x\|_2 \le M\,\|x\|_1$$

for every x in N

(b) Let N and N' be normmed linear spaces and let T: N⟶N' be any linear transformation. If N finite. dimensional then T is continuous (or bounded)

(c) Let N be an arbitrary normed linear space. Then each vector x in N induces a functional F_x on N* defined by

$$F_x\,(f) = f(x)\ \forall\, f \in N^*$$

Such that $\|F_x\| = \|x\|$

Further the mapping J: N ⟶ N** : J(x) = F_x $\forall\, x \in$N

defines an isometric isomorphism of N into N**

Q.5. (a) Show that $\ell_p^* = \ell_p$ where $\frac{1}{p}+\frac{1}{q}=1$ and $1< P < \infty$

(b) State and prove the uniform bounded theorem.

(c) Let T be am operator on a normed linear space N, then its conjugate T*, defined by

$$T^* : N^* \to N^* : T^*(f) = foT \text{ and}$$

$$[T^*(f)](x) = f(T(x) \text{ for all } f \in N^* \ \&\ x \in N$$

is an operator on N* and the mapping

$$\emptyset : B(N) \to B(N^*) : \emptyset(T) = T^*\ \forall\, T \in B(N)$$

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