Total No. of Questions : 05] [SET-A] [Total No. of Printed Page : 02

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

Final Year – Third Semester

Advance Discrete Mathematics

3M.Sc. 4

Time : 3 Hours **Max Marks : 70**

Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) Define algebraic structure and prove that $G = \{1, w, w^2\}$ is a group with respect to the multiplication, where $1, w, w^2$ are cube roots of unity.

(b) Define Group Homomorphism with example. Also show that if f be a homomorphism of $(G,*)$ into $(\bar{G},.)$ with kernel K, then K is a normal Subgroup of G.

(c) State and prove "Fundamental theorem of Homomorphism of Groups"

Q.2. (a) Define semi group and show that if g is a homomorphism from a commutative semi group $(S,*)$ onto a semigroup $(T,\oplus)$ then $(T,\oplus)$ is also a commutative semigroup.

(b) Prove that every finite semigroup has an idempotent element.

(c) Define monoid with example. Also show that if $(m,*,e)$ and (T,Δ,e') be two monoids with identities e and e' if f is an onto mapping from M to T i,e $f: M \to T$ is a isomorphism.

Q.3. (a) Define lattice with example. Also show that if $(L,\leq)$be a lattice, then for any $a, b, c \in L$

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(b) Show that dual of a lattice is a lattice.

(c) Let $(L,\leq)$ be a lattice and $a, b, c, d \in L$ then the following implications hold:

(i) $a \leq b \text{ and } c \leq d \Longrightarrow a \vee c \leq b \vee d$

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(ii) $a \leq b \text{ and } c \leq d \Rightarrow a \wedge c \leq b \wedge d$

Q.4. (a) Define complemented lattice and show that two bounded lattices L_1 and L_2 are complemented iff $L_1 \times L_2$ is complemented

(b) In a Boolean algebra $(B, +, ., ')$ prove the following.

(i) $a.b + b.c + c.a = (a+b).(b+c).(c+a)$

(ii) $[a + (a' + b)'].[a + (b'.c)'] = a$

(c) If S_1 and S_2 are two sub algebra of a Boolean algebra B then $S_1 \cap S_2$ is also a sub algebra of B.

Q.5. (a) Show that a tree T with n vertices has n-1 edges.

(b) Prove that every tree has either one or two centres.

(c) Define spanning tree. Also prove that "Every connected graph has at least one spanning tree"

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