Total No. of Questions : 05] **[SET-A]** **[Total No. of Printed Page : 02**

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

First Year – Second Semester

Real Analysis – II

2M.Sc. 2

Time : 3 Hours **Max Marks : 70**

Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) Let f be a bounded and g a non decreasing function on [a,b]. Then prove that $f \epsilon RS(g)$ if and only if for every $\varepsilon > o$ there exists a partition P such that $U(p, f, g) - L(P, f, g) < \varepsilon$

(b) let $f \,\epsilon\, RS(g)$ on [a,b]. Than prove that $m[g(b) - g(a)] \leq \int_a^b f dg \leq M[g(b) - g(a)]$ where m, M are the bounds of f on [a,b].

(c) Let let $f \,\epsilon\, RS(g)$ on [a,b]. if a<c<b, then show that $f \,\epsilon\, RS(g)$ on [a,c], $f \,\epsilon\, RS(g)$ on [c,b] and $\int_a^b f dg = \int_a^c f dg + \int_c^b f dg$.

Q.2. (a) Let f be a mapping of [a,b] into R^P and let $f \,\epsilon\, RS(g)$ on [a,b].for some monotonically non – decreasing function g on [a,b]. then prove that $|f| \,\epsilon\, RS(g)$ on [a,b]. and $\left|\int_a^b f dg\right| \leq \int_a^b |f| dg$.

(b) Let f and g be complex valued function of bounded variation on [a,b], and let f be also continuous on [a,b]. Then prove that -

$$\int_a^b f dg = f(b)g(b) - f(a)g(a) - \int_a^b g df.$$

(c) Let f and ∅ be continuous on [a,b] and let ∅ be increasing on [a,b]. if F is the inverse function of ∅ , then prove that -

$\int_a^b f(x)dx = \int_{\emptyset(a)}^{\emptyset(b)} f\big(F(y)\big)\, dF\,(y)$.

Q.3. (a) Show that the function

$f(x, y) = \frac{xy}{\sqrt{(x^2+y^2)}}, x \neq 0, y \neq 0 \; and \; f(0,0) = 0$ is continuous at the origin in (x,y) together.

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(b) let $f(x,y) = x\sin\frac{1}{x} + y\sin\frac{1}{y}, x \neq 0, y \neq 0; f(0,y) = y\sin\frac{1}{y}, y \neq 0;$ $f(x,0) = x\sin\frac{1}{x}, x \neq 0$ and $f(0,0) = 0$ Examine the existence of f_x and f_{yx} at $x = 0, y = 0$.

(c) If $= \sin(y/x)$, where $x = 3r^2 + 3s, y = 4r - 2s^3$ $Z = 2r^2 - 3s^2$, find, $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$.

Q.4. (a) Suppose f is continuous mapping of [a,b] into R^n and f is differentiable in]a,b[then prove that there exists $x_o \epsilon]a,b[$ such that $|f(b) - f(a)| \leq (b-a)|f'(x_o)|$.

(b) Suppose A is an open subset of R^n, f maps A into R^m, f is differentiable at $x_o \epsilon A$, g map an open set containing $f(A)$ into R^k, and g is differentiable at $f(x_o)$. Then show that the mapping F of A into R^k defined by $F(x) = g(f(x))$ is differentiable at x_o and $F'(x_o) = g'(f(x_o)f'(x_0)$

(c) Let A be an open subset of R^{n+m} and le f be a C^{-1}mapping of A into R^n, If $(a,b)\epsilon A, f(a,b) = 0, T = f'(a,b)$ and $f(h,o) = 0$ implies h=0, then prove that there exists a neighbourhood w of b ($W \subset R^m$)and a unique function $g\epsilon c'(w)$, with values in R^n, such that g (b) = a and $f(g(y),y) = 0$; $y\epsilon W$

Q.5. (a) Find the Jacobean of $y_1, y_2, y_3, --- y_n$, being given $y_1 = x_1(1-x_2)$, $y_2 = x_1x_2(1-x_3), \ldots y_{n-1} = x_1x_2\ldots x_{n-1}(1-x_n), y_n = x_1x_2\ldots x_n$

(b) If $u_1, u_2, \ldots u_n$ are functions of $y_1, y_2 \ldots y_n$ and $y_1, y_2 \ldots y_n$ are function of $x_1, x_2, \ldots x_n$, then prove that

$$\frac{\partial(u_1,u_2,\ldots u_n)}{\partial(x_1,x_2,\ldots x_n)} = \frac{\partial(u_1,u_2,\ldots u_n)}{\partial(y_1,y_2,\ldots y_n)}, \frac{\partial(y_1,y_2,\ldots y_n)}{\partial(x_1,x_2,\ldots x_n)}$$

(c) State and prove the necessary and sufficient condition for a Jacobian to vanish identically.

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