Total No. of Questions : 05] [SET-A] [Total No. of Printed Page : 02

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

First Year – Second Semester

Complex Analysis – II

2M.Sc. 4

Time : 3 Hours **Max Marks : 70**

Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) Determine the Singularity of function

$$f(z) = \begin{cases} \frac{sinz}{z} \; ; \; z \neq 0 \\ o \; ; \; z = 0 \end{cases}$$

(b) What kind of singularity have the following function:

(i) $f(z) = sin\frac{1}{1-z} \; at \; z = 1$

(ii) $f(z) = sinz - cosz \; at \; z = \infty$

(c) State and proof uniqueness theorem.

Q.2. (a) Define Residue at infinity and find the residue of

$f(z) = \frac{z^3}{z^2-1} \; at \; z = \infty$

(b) State and proof Cauchy Residue's theorem.

(c) Find residue of the function:

$$f(z) = \frac{cos\Pi z}{(z-a)^2}$$

Q.3. (a) Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\Pi}{3}$

(b) Evaluate $\int_{o}^{\infty} \frac{cos^2 x}{(1+x^2)^2} \, dx$

(c) Prove that $\int_{o}^{\infty} \frac{dx}{x^4+1} = \frac{\Pi\sqrt{2}}{4}$

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Q.4. (a) If $\alpha \, and \, \beta$ are two given points and K is a constant, show that the equation. $\left|\frac{z-\alpha}{z-\beta}\right| = K$ represents a circle.

(b) Find bilinear transformation which maps $z = 1, i, -1$ respectively onto $W = i, o, -i.$

(c) Find normal form of the bilinear transformation and discuss its nature also;

$W = \frac{3iz+1}{Z+i}$

Q.5. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not Analytic of origin, although Cauchy-Riemann equation are satisfied at the point.

(b) Show that the function $f(z) = \bar{z}$ is not analytic.

(c) Prove that $u = e^{-x}(x \, siny - ycosy)$ is harmonic.

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