Total No. of Questions : 05] **[SET-A]** **[Total No. of Printed Page : 02**

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

First Year – Second Semester

Advance Abstract Algebra – II

2M.Sc. 1

Time : 3 Hours **Max Marks : 70**

Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) The Submodule of the quotient module M/N is of the form U/N, where U is a Submodule of M containing N.

(b) State & prove fundamental theorem of homomorphism of modules.

(c) Prove that every abelian group G is a module over the ring of integers Z.

Q.2. (a) State and prove first isomorphism theorem.

(b) Find the abelian group generated by (x_1, x_2, x_3) subject to

$5x_1 + 9x_2 + 5x_3 = 0$

$2x_1 + 4x_2 + 2x_3 = 0$

$x_1 + x_2 + 3x_3 = 0$

(c) Let R be a PID be a unity. Let a, b $\in R^*$. Then d = gcd (a,b) $\Leftrightarrow d =$ (a)+(b).

Q.3. (a) Prove that every finitely generated module M = $(x_1,x_2,\ldots\ldots,x_n)$ is a homomorphic image of a finitely generated free module R^n.

(b) Every submodule and every quotient module of noetherian module is noetherian.

(c) Every homomorphic image of an artinian module is also artinian.

Q.4. (a) State and prove Hilbert basis theorem.

(b) Let A be minimal left ideal in a ring R. Then either $A^2 = (0)$ or A= Re, where e is an idempotent in R.

(c) State and prove Wedder burn-Artin Theorem.

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