Total No. of Questions : 05] [SET-A] [Total No. of Printed Page : 02

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

First Year – First Semester

Topology – I

1M.Sc. 3

Time : 3 Hours **Max Marks : 70**

Min. Marks : 25

Note :- Solve any two parts from each question. All questions carry Equal marks.

Q.1. (a) Define topological space. Also prove that the set of all open balls in a metric space form a topological space.

(b) Define topological space in terms of closed sets. Also prove that $\bar{A}$ (the closure of A) is the smallest closed subset of topological space X containing A.

(c) Prove that a subset A of X is dense in X iff every non empty open subset B of X, $A \cap B \neq \emptyset$.

Q.2. (a) Prove that a the interior of a set is same as the complement of the closure of the complement of the set i.e. for a subset A of space X int (A) = X- $(\overline{X-A})$

(b) Let $(X, \mathcal{J})$ be a topological space and $\mathcal{B} \subset \mathcal{J}$. Then prove that $\mathcal{B}$ is a base for $\mathcal{J}$ iff for any $x \epsilon X$ and any open set G containing x, there exist $B \epsilon \mathcal{B}$ such that $x \epsilon B$ and $B \subset G$.

(c) Let X be a space and $A \subset X$, Then int (A) is the union of all open subset of X contained in A. It is also the largest open subset of X contained in A.

Q.3. (a) Let $(X, \mathcal{J})$ and (Y,U) be two topological spaces. A mapping f: X→Y is continuous iff the inverse image of every open set in Y is open in X.

(b) Prove that the composition of continuous functions are continuous.

(c) Suppose f:X→Y is continuous at a point $x_0 \in X$. Prove that whenever a sequence $\{x_n\}$ converges to x_o in X, the sequence $\{f(x_n)\}$ converges to $f(x_o)$ in Y.

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Q.4. (a) Let G be the collection of connected subsets of a space X such that no two members of C are mutually separated. Then $\underset{c \in C}{U\ C}$ is also connected.

(b) Prove that for a topological space following are equivalent.

(i) The space X is T_1- space

(ii) For any x∈X, the singleton set {x}is closed.

(iii) Every finite subset of X is closed.

(iv) The topology $\mathcal{J}$ is stronger than the cofinite topology on X.

(c) Prove that, in a Hausdorff space, limits of sequence are unique.

Q.5. (a) Prove that every second countable space is separable

(b) Prove that every continuous real valued function on a compact space is bounded and attains its extreme.

(c) Prove that every continuous image of a compact space is compact.

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