

FINAL EXAMINATION – JULY 2017  
MASTER OF SCIENCE (M.Sc. MATHEMATICS)

First Year – First Semester  
Complex Analysis – I

1M.Sc. 4

Time : 3 Hours

Max Marks : 70  
Min. Marks : 25

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Note :- Solve any two parts from each question. All questions carry Equal marks.

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- Q.1. (a) State and prove C-R- equation in polar form.  
(b) Find an analytic function whose imaginary part is given by  $\tan^{-1}\left(\frac{y}{x}\right)$  and hence find the real part.  
(c) Define modulus and argument of a complex no. and find it for  $Z = \frac{1+2i}{1-(1-i)^2}$
- Q.2. (a) Prove that  $\frac{\partial(u,v)}{\partial(x,y)} \frac{\partial(x,y)}{\partial(u,v)} = 1$ .  
(b) Consider the transformation  $w = 2z$  and determine the region D' of the w-plane into which the triangular region D enclosed by the lines  $x = 0, y = 0, x + y = 1$  in the z-plane is mapped under this transformation.  
(c) Find the bilinear transformation that maps the points  $z_1 = \infty, z_2 = i, z_3 = 0$  into the points  $w_1 = 0, w_2 = i, w_3 = \infty$ .
- Q.3. (a) Evaluate  $\int (\bar{z})^2 dz$  around the circle  $|z - 1| = 1$ .  
(b) Evaluate  $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$  where  $c: |z|=3$   
(c) Let  $f(z)$  be analytic in a simple closed region R. Then show that the integral of  $f(z)$  along any closed curve in R is path independent.
- Q.4. (a) State and prove Cauchy integral formula for simply connected region.

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- Q.4. (a) State and prove Cauchy integral formula for simply connected region.

(b) Using Cauchy's integral formula; calculate

$$\int_C \frac{z dz}{(9-z^2)(z+i)} ; c: |z| = 2$$

(c) State and prove Liouville's theorem.

Q.5. (a) State and proof Laurent's theorem.

(b) Expand  $f(z) = \frac{1}{z^2-3z+2}$  for  $0 < |z| < 1$

(c) Expand  $f(z) = \sin z$  in a Taylor series about  $z = \frac{\pi}{4}$

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