Total No. of Questions : 05] **[SET-A]** **[Total No. of Printed Page : 02**

FINAL EXAMINATION – JULY 2017

MASTER OF SCIENCE (M.Sc. MATHEMATICS)

First Year – First Semester

Advance Abstract Algebra – I

1M.Sc. 1

Time : 3 Hours **Max Marks : 70**

Min. Marks : 25

Note :- **Solve any two parts from each question. All questions carry Equal marks.**

Q.1. (a) Every finite group has a composition series.

(b) State and prove Jordan-Holder theorem for finite groups.

(c) Show that an abelian group G has a composition if and only if G is finite.

Q.2. (a) Every subgroup of a solvable group is solvable.

(b) A group G is nilpotent if and only if G has a normal series
$(e) = G_0 \subset G_1 \subset G_2 \subset \ldots\ldots \subset G_n = G$.
Such that $G_i/G_{i-1} \subset Z\left({G}/{G_{i-1}}\right)$ for all i = 1,2,…..,m.

(c) Let H be a normal subgroup of G. If both H & G/H are solvable, then G is also solvable.

Q.3. (a) Let F $\subseteq$ E $\subseteq$ K be fields. If K is a finite extension of E and E is a finite extension of F, then K is a finite extension of F and [K:F] = [K:E] [E:F].

(b) Every finite extension of a field is an algebraic extension.

(c) Let Q be the field of rationals then show that $Q\left(\sqrt{2},\sqrt{3}\right) = Q\left(\sqrt{2}+\sqrt{3}\right)$.

Q.4. (a) State and prove Lagrange's Theorem.

(b) Let G be a finite group. The number of elements conjugate to a in G is the index of normalize of a in G i.e.

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Where, $C_a = 0(C(a))$.

(c) State and prove Cauchy's Theorem for abelian group.

Q.5. (a) If f(x) and g(x) are two non-zero polynomials members of R[x] Then

(i) Deg [f(x) + g(x)] $\leq$ max [deg f(x), deg g(x)], if f(x) + g(x) $\neq$ 0.

(ii) Deg [f(x).g(x)] $\leq$ [deg f(x) + deg g(x)]

R[x] represent the set of polynomials over a ring.

(b) The set R[x] of all polynomials over an arbitrary ring (R,+.) is a ring with respect to addition and multiplication of two polynomials.

(c) Let R be a unique factorization domain. Then every non-zero member f(x) of R[x] is expressible as a product $af_1(x)$ where a = c(f) and $f_1(x)$ is a primitive member of R [x] and this expression is unique part from the differences in associates.

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