

Course Code: 3MSCM3
 Course: Special Functions-I
 Credit: 4
 Last Submission Date: April 30 (for January Session)
 October 31, (for July session)

Max. Marks:-70
 Min. Marks:-25

Note:-attempt all questions.

Que.1 Prove that $\Gamma(z) = \lim_{n \rightarrow \infty} \int_0^n \left(1 - \frac{t}{n}\right)^n z^{-1} dt$

Que.2 State & prove Gauss multiplication theorem.

Que.3 Prove that if $\operatorname{Re}(c-a-b) > 0$ & $\operatorname{Re}(c) > \operatorname{Re}(b) > 0$ and c is neither zero nor a negative integer then

$$F(a, b, c, 1) = \frac{\Gamma(c) \Gamma(c-a-b)}{\Gamma(c-a) \Gamma(c-b)}$$

Que.4 The complete elliptic integral of first kind being

$$K = \int_0^{1/2\pi} \frac{d\phi}{\sqrt{1-k^2 \sin^2 \phi}} \text{ . Show that } k = \frac{1}{2} \pi F\left(\frac{1}{2}, \frac{1}{2}; 1; k^2\right)$$

Que.5 State & prove Whipple's theorem.

Que.6 State & prove Saalschut's theorem.

Que.7 State & prove Kummer's theorem.

Que.8 State & prove Ramanujan's theorem.

Que.9 Prove that

$$(1) \frac{d}{dz} [z^n J_n(z)] = z^n J_{n-1}(z)$$

$$(2) z J_n'(z) = z J_{n-1}(z) - n J_n(z)$$

Que.10 Prove that

$$J_{\frac{3}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \left[\frac{1}{x} \sin x - \cos x\right]$$