Course Code: 3MSCM4 Course: Advanced Discrete mathematics (Elective-I) Credit: 4 Last Submission Date: April 30 (for January Session) October 31, (for July session)

> Max. Marks:-70 Min. Marks:-25

Note:-Attempt all questions.

- Que.1 Consider a set I of integers. Let (I,+,X) be the algebraic system, where + and x are the operation of addition and multiplication on I.
- Que.2 Define homomorphism & isomorphism with example.
- Que.3 Let (s,*) and (T, \bigoplus) be two semigroup and f:S \rightarrow T be a semigroup homomorphism c then corresponding to f $\exists a$ congruence relation R on (s,*) be defined by aRb iff $f(a) = f(b) \forall a, b \in S$
- Que.4 Let (M, *, e) and (T, Δ, e') be two monoids with identifies e and e' if f is an onto mapping from M to T i.e. f: $M \rightarrow T$ is an isomorphism then f(e) = e'.

Que.5 Let
$$(L, \leq)$$
 be a lattice. Then the following results hold:

(A) For each
$$a \in L$$
 then
 a_1) $a \wedge a = a$
 b_2) $a \vee a = a$
(B) For any $a, b, \in L$ then
 b_1) $a \wedge b = b \wedge a$
 b_2) $a \vee b = b \vee a$
(C) For any $a, b, c \in L$
 c_1) $(a \wedge b) \wedge c = a \wedge (a \wedge c)$
 c_2) $(a \vee b) \vee c = a \vee (b \vee c)$
(D) For $a, b \in L$ then
 d_1) $a \wedge (a \vee b) = a$
 d_2) $a \vee (a \wedge b) = a$

Que.6 In any lattice (L, \land, \lor) the following statement are equivalent:

(1)
$$a \land (b \lor c) = (a \land b) \lor (a \land c) \forall a, b, c \in L$$

(2)
$$a \lor (b \land c) = (a \lor b) \land (a \lor c) \forall a, b, c \in L$$

- Que.7 State & prove Demorgan's low in Boolean algebra.
- Que.8 In a Boolean algebra (B,+, ., .) then show that a+b=a+c and ab=ac then b=c
- Que.9 A tree T with n-vertices has n-1 edges.
- Que.10 To prove that every connected graph has at least one spanning tree.