## Course Code: 3MSCM4

Course: Advanced Discrete mathematics (Elective-I)
Credit: 4
Last Submission Date: April 30 (for January Session)
October 31, (for July session)
Max. Marks:-70
Min. Marks:-25
Note:-Attempt all questions.
Que. 1 Consider a set I of integers. Let ( $\mathrm{I},+, \mathrm{X}$ ) be the algebraic system, where + and x are the operation of addition and multiplication on I.
Que. 2 Define homomorphism \& isomorphism with example.
Que. 3 Let $(\mathrm{s}, *)$ and $(\mathrm{T}, \oplus)$ be two semigroup and $\mathrm{f}: \mathrm{S} \rightarrow \mathrm{T}$ be a semigroup homomorphism $c$ then corresponding to $\mathrm{f} \exists a$ congruence relation R on ( $\mathrm{s}, *)$ be defined by aRb iff $\mathrm{f}(\mathrm{a})=\mathrm{f}(\mathrm{b}) \forall \mathrm{a}, \mathrm{b} \in \mathrm{S}$
Que. 4 Let ( $\mathrm{M}, *, e$ ) and (T, $\Delta, e^{\prime}$ ) be two monoids with identifies e and e' if f is an onto mapping from M to T i.e. $\mathrm{f}: \mathrm{M} \rightarrow \mathrm{T}$ is an isomorphism then $\mathrm{f}(\mathrm{e})=\mathrm{e}^{\prime}$.
Que. 5 Let ( $\mathrm{L}, \leq$ ) be a lattice. Then the following results hold:
(A) For each $a \in L$ then
$\left.a_{1}\right) a \wedge a=a$
$b_{2}$ ) $a v a=a$
(B) For any $a, b, \in L$ then
$\left.b_{1}\right) a \wedge \mathrm{~b}=\mathrm{b} \wedge \mathrm{a}$
$b_{2}$ ) $a \vee b=b \vee a$
(C) For any $a, b, c \in L$

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\begin{aligned}
& \left.c_{1}\right)(a \wedge \mathrm{~b}) \wedge \mathrm{c}=\mathrm{a} \wedge(\mathrm{a} \wedge \mathrm{c}) \\
& \left.c_{2}\right)(a \vee b) \vee c=a \vee(b \vee c)
\end{aligned}
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(D) For $a, b \in L$ then
$\left.d_{1}\right) a \wedge(a \vee b)=a$
$\left.d_{2}\right) \quad a \vee(a \wedge b)=a$
Que. 6 In any lattice ( $\mathrm{L}, \wedge, \mathrm{v}$ ) the following statement are equivalent:
(1) $a \wedge(b \vee c)=(a \wedge b) \vee(a \wedge c) \forall a, b, c \in L$
(2) $a \vee(b \wedge c)=(a \vee b) \wedge(a \vee c) \forall a, b, c \in L$

Que. 7 State \& prove Demorgan's low in Boolean algebra.
Que. 8 In a Boolean algebra ( $B,+, .,{ }^{\prime}$ ) then show that $a+b=a+c$ and $a b=a c$ then $b=c$
Que. 9 A tree T with n - vertices has $\mathrm{n}-1$ edges.
Que. 10 To prove that every connected graph has at least one spanning tree.

